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BIHARMONIC HOPF CYLINDERS

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0. Introduction. This paper concerns curves and surfaces, into indefinite space forms, whose mean curvature vector field is in the kernel of certain elliptic differential operators. It has been inspired by the paper of M. Barros and O.J. Garay [**2**], where the Riemannian version of this question is solved. We first consider the Laplacian to study indefinite submanifolds with harmonic mean curvature vector field in the normal bundle. This problem is closely related to a conjecture of B.-Y. Chen [**5**], on Riemannian submanifolds, stated as follows: harmonicity of the mean curvature vector field implies harmonicity of the immersion. Submanifolds with harmonic mean curvature vector field were called by Chen *biharmonic submanifolds*. In the realm of indefinite submanifolds, counterexamples to that conjecture have been given by the two first authors, see [**1**]. Biharmonic submanifolds are a special class of submanifolds for which its mean curvature vector is an eigenvector of Δ, that is, $\Delta H = \lambda H$ for some real constant λ. First we describe the family of curves whose mean curvature vector field is proper for the Laplacian. This problem has been solved for Euclidean curves by M. Barros and O.J. Garay [**2**]. We have to think of a different Laplacian if we want to characterize curves others than those of both constant curvature and torsion. Since H is a normal vector field, it seems natural to consider the Laplacian associated to the connection in the normal bundle. Then we show that the indefinite Cornu spirals are the only nonstandard curves in a semi-Riemannian manifold that are biharmonic in the normal bundle. As for surfaces, we deal with the semi-Riemannian Hopf cylinders we introduced in [**4**]. Then we show that the biharmonicity of them strongly depends on the biharmonicity of the curves to which they are associated. In fact, a nonstandard Hopf cylinder in $\mathbf{H}^3_1(-1)$ is biharmonic in the normal bundle if and only if it is associated to a Cornu spiral in $\mathbf{H}^2_s(-4)$. Then we extend the results in [**2**].

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