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BALL SEPARATION PROPERTIES IN BANACH SPACES

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Dedicated to Professor Ky Fan on the occasion of his 85th birthday

ABSTRACT. Various ball separation properties related to Mazur intersection property in Banach spaces are studied.

Mazur [**16**] was the first to consider the following ball separation property, called property (I), in Banach spaces.

(I) Every bounded closed convex set is an intersection of (closed) balls.

For finite dimensional Banach space X, Phelps [**17**] showed that X has the property (I) if and only if the set of extreme points of the unit ball $B(X^*)$ of the dual space X^* is norm dense in the unit sphere $S(X^*)$ of X^*.

Giles, Gregory and Sims [**10**] showed that a Banach space X has the property (I) if and only if the set of weak* denting point of $B(X^*)$ is norm dense in $S(X^*)$. They raised a question whether every Banach space with the property (I) is an Asplund space. In 1995, Sevilla and Moreno [**20**] exhibit a class of non-Asplund spaces admitting an equivalent norm with property (I). It has been proved recently by Jimenez and Moreno [**14**] that Kunen space is an Asplund space with no equivalent norm with property (I).

Whitefield and Zizler [**21**] studied the following ball separation property, called (CI).

(CI) Every compact convex set is an intersection of balls.

They proved that a Banach space X has the property (CI) if the cone generated by the extreme points of $B(X^*)$ is τ_X dense in X^*

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