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THE EXISTENCE OF SHAPE-PRESERVING OPERATORS WITH A GIVEN ACTION

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ABSTRACT. We study the existence of shape-preserving projections and, more generally, the existence of shape-preserving operators with a given (fixed) action.

Introduction and preliminaries. Let X denote a Banach space and V an n-dimensional subspace of X. We will use the following notation. An n-tuple from X is to be considered a column vector while an n-tuple from X^* will be a row vector. Elements of $\mathbf{R}^n$ will be column vectors.

Let $S \subset X$ denote the set of all elements that possess a specified "shape." For example, S might denote the set of convex functions or the set of monotone functions in $C[0,1]$. The problems involved with preserving the "shape," i.e., leaving S invariant, while approximating elements of X by elements of V have been the object of much study, especially in the case of best approximation (see, for example, [**2, 4, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19**]). Best approximation operators that are invariant on S are, in general, nonlinear and their existence is usually not an issue. It is in the attempt to preserve a "shape" using *linear* operators that existence becomes problematic. As illustrated in the following example, small variations in the "action" of a linear operator on V may greatly influence the ability of that operator to leave S invariant.

Example 1.1. Let Π_2 denote the space of second-degree algebraic polynomials, considered as a subspace of $C[0,1]$. The second-degree Bernstein operator $B_2 : C[0,1] \to \Pi_2$ is a linear operator that preserves monotonicity. This is accomplished while *nearly fixing* Π_2 (B_2 fixes the lines and $B_2 t^2 = (t+t^2)/2$). However, no linear operator *fixing* Π_2 can preserve monotonicity. Indeed, if such an operator $P : C[0,1] \to \Pi_2$ did exist, we could rewrite it as $P = \sum_{i=1}^3 u_i \otimes t^{i-1}$ where

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