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ASYMPTOTIC FORMULAE OF LIOUVILLE-GREEN TYPE FOR A GENERAL FOURTH-ORDER DIFFERENTIAL EQUATION

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ABSTRACT. As asymptotic form of solutions of Liouville-Green type for a general fourth-order differential equation are given under general conditions on the coefficients for large x.

1. Introduction. In this paper we consider the asymptotic form of four linearly independent solutions of a general fourth-order differential equation

$$(1.1)\quad (p_0y'')'' + (p_1y')' + \frac{1}{2}\sum_{j=0}^{1}[\{q_{2-j}\cdot y^{(j)}\}^{(j+1)} + \{q_{2-j}\cdot y^{(j+1)}\}^{(j)}] - p_2y = 0$$

as $x \to \infty$, where x is the independent variable and the prime denotes d/dx. The functions p_j, $1 \leq j \leq 3$, and q_j, $j = 1, 2$, are defined on an interval $[a, \infty)$ and are not necessarily real-valued, while p_0 is nowhere zero in this interval. We shall consider the case where the three functions $q_1(p_2/p_0)^{3/4}$, $p_1(p_2/p_0)^{1/2}$ and $q_2(p_2/p_0)^{1/4}$ are all small compared to p_2 as $x \to \infty$.

In this case the solutions all have a similar exponential factor as given below in Theorem 4.1.

In the case where $p_1 = q_1 = q_2 = 0$, (1.1) reduces to

$$(1.2)\qquad (p_0y'')'' - p_2y = 0$$

which is the case $n = 4$ of the nth order equation considered by Hinton [**9**] and Eastham [**4**], and they showed that, subject to certain conditions in the coefficients p_0 and p_2, (1.2) has solutions

$$(1.3)\qquad y_k(x) \sim p_0^{-1/8}(x)p_2^{-3/8}(x)\exp\left(\omega_k\int_a^x \left(\frac{p_2}{p_0}\right)^{1/4}(t)\,dt\right)$$

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