ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 28, Number 3, Fall 1998

DESCRIPTION OF A CLASS OF LOCALLY PSEUDOCONVEX ALGEBRAS WHICH HAVE AN EQUIVALENT LOCALLY M-PSEUDOCONVEX TOPOLOGY

MATI ABEL AND JORMA ARHIPPAINEN

ABSTRACT. Let $r \in (0,1]$, p be an r-homogeneous semi-norm on a commutative algebra A and $\phi : [0,\infty) \mapsto [0,\infty)$ be an increasing function such that $p(x^2) \leq \phi(p(x))$ for all $x \in A$. It is shown that such a semi-norm p is (i) A-pseudoconvex, (ii) equivalent with a submultiplicative semi-norm. A description of a class of locally pseudoconvex algebras which have an equivalent locally m-pseudoconvex topology is given.

Introduction. Let A be a topological algebra, that is, a linear topological space over $\mathbf{C}$ in which has been defined a separately continuous (associative) multiplication. If the underlining linear topological space A is locally pseudoconvex, then A is called a locally pseudoconvex algebra. It is known (see [**11**, p. 6]) that the topology of locally pseudoconvex algebras can be given by means of a family $\mathcal{P} = \{p_\lambda \mid \lambda \in \Lambda\}$ of r_λ-homogeneous semi-norms where $r_\lambda \in (0,1]$ is fixed for each $\lambda \in \Lambda$ (r-homogeneity of a semi-norm p on A means that $p(\alpha x) = |\alpha|^r p(x)$ for each $x \in A$ and $\alpha \in \mathbf{C}$). Furthermore, A is called a locally absorbingly pseudoconvex (shortly a locally A-pseudoconvex) algebra if, for each $x \in A$ and $\lambda \in \Lambda$, there exist positive numbers $M_\lambda(x)$ and $N_\lambda(x)$ such that

$$p_\lambda(xy) \leq M_\lambda(x) p_\lambda(y) \tag{1}$$

and

$$p_\lambda(yx) \leq N_\lambda(x) p_\lambda(y) \tag{2}$$

for all $y \in A$. In particular, if $M_\lambda(x) = N_\lambda(x) = p_\lambda(x)$ for each $x \in A$ and $\lambda \in \Lambda$, then A is called a locally multiplicatively pseudoconvex (shortly a locally m-pseudoconvex) algebra. Moreover, if the numbers $M_\lambda(x)$ and $N_\lambda(x)$ do not depend on semi-norms p_λ, $\lambda \in \Lambda$, i.e., (1)

Received by the editors on April 12, 1996.