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SOME REMARKABLE CONGRUENCES ON COMPLETELY REGULAR SEMIGROUPS

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ABSTRACT. We express a completely regular semigroup S as $(Y; S_\alpha)$, that is, a semilattice of completely simple semigroups. For each pair $\alpha > \beta$, we consider the congruence $\kappa_{\alpha,\beta}$ on S generated by the set of pairs (a, b) where $a \in S_\alpha$, $b \in S_\beta$ and $a > b$. These congruences play an important role in finding conditions which ensure that the kernel relation K on the congruence lattice of S be a congruence. In particular, the meet and the join of these congruences provide interesting congruences in this context. Another class of congruences, constructed as follows, occurs naturally in this study. Given a congruence ρ on S and ideals $I \subseteq J$ of S, we generalize the Rees congruence relative to I by constructing a congruence which involves ρ, I and J; here ρ must saturate I and I or J may be empty.

1. Introduction and summary. The consideration of necessary and sufficient conditions on a completely regular semigroup S in order that the kernel relation K on the congruence lattice $\mathcal{C}(S)$ be a congruence in [**5**] gives rise to the following class of congruences. We write $S = (Y; S_\alpha)$ thereby indicating that S is a semilattice Y of completely simple semigroups S_α. For each pair $\alpha, \beta \in Y$ such that $\alpha > \beta$, let $\kappa_{\alpha,\beta}$ be the congruence on S generated by the pairs (a, b) such that $a \in S_\alpha$, $b \in S_\beta$, $a > b$. These congruences play a crucial role in the above evoked study. Besides the conditions on S which ensure that K be a congruence, it is of interest to find some lattices Λ of congruences on an arbitrary completely regular semigroup S with the property that $K|_\Lambda$ is a congruence.

Section 2 contains the minimum of necessary preliminaries. We establish in Section 3 that K restricted to the filter of $\mathcal{C}(S)$ generated by the join of congruences $\kappa_{\alpha,\beta}$ is a congruence and the corresponding

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