

COINCIDENCE PRINCIPLES AND
FIXED POINT THEORY FOR
MAPPINGS IN LOCALLY CONVEX SPACES

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ABSTRACT. We present a coincidence principle for concentrative maps. This leads to new fixed point theory for nonlinear operators.

1. Introduction. A general coincidence theory is presented for concentrative mappings between locally convex Hausdorff linear topological spaces in this paper. These general results are used to obtain a variety of new fixed point theorems for the sum of two operators, for example, an m -accretive plus a condensing operator, between Banach spaces (one could also obtain results for operators between locally convex Hausdorff linear topological spaces). The fixed point results were motivated from a variety of sources, in particular we mention the work of Browder [4], Daneš [7], Furi and Pera [14], Gatica and Kirk [15], Granas [16], Petryshyn [25], Precup [26], Reiner mann [27] and Schöneberg [28]. Some applications of our results are also presented in this paper.

For the remainder of this section we gather together some definitions and some known facts. Let (E, d) be a pseudometric space [18] and M a subset of E . For $x \in M$, let $B(x, \varepsilon)$ denote the closed ε -ball with center x , i.e., $B(x, \varepsilon) = \{y \in E : d(x, y) \leq \varepsilon\}$. The *measure of noncompactness* of the set M is defined by

$$\alpha(M) = \inf Q(M); \quad \inf(\emptyset) = \infty,$$

where

$$Q(M) = \{\varepsilon \in \mathbf{R} : \varepsilon > 0 \text{ and there is a finite } \varepsilon\text{-net for } M \text{ in } E \\ \text{i.e., } M \subseteq B(A, \varepsilon) \text{ for some finite subset } A \text{ of } E\}.$$

Note $B(A, \varepsilon) = \{x \in E : \inf\{d(x, y) : y \in A\} \leq \varepsilon\}$.

Received by the editors on September 10, 1995.

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