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GENERALIZED BERNSTEIN-CHLODOWSKY POLYNOMIALS

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ABSTRACT. For given positive integers n and m, the following generalization of Bernstein-Chlodowsky polynomials is studied,

$$B_{n,m}(f,x) = (1+(m-1)\frac{x}{b_n}) \sum_{k=0}^{[n/m]} f\left(\frac{b_n k}{n-(m-1)k}\right) \cdot C_{n-(m-1)k}^k \left(\frac{x}{b_n}\right)^k \left(1-\frac{x}{b_n}\right)^{n-mk},$$

where b_n is a sequence of positive numbers such that $\lim_{n\to\infty} b_n = \infty$, $\lim_{n\to\infty}(b_n/n) = 0$, $0 \le x \le b_n$ and $[p]$, as usual, denotes the greatest integer less than p. A theorem about convergence of $B_{n,m}(f,x)$ to $f(x)$ as $n \to \infty$ in weighted space of functions f continuous on positive semiaxis and satisfying the condition $\lim_{x\to\infty}(f(x)/(1+x^2)) = K_f < \infty$ is established.

1. Let $\rho(x) = 1+x^2$, $-\infty < x < \infty$ and B_ρ be the set of all functions f defined on the real axis and satisfying the condition $|f(x)| \le M_f \rho(x)$ with some constant M_f, depending only on f. By C_ρ we denote the subspace of all continuous functions belonging to B_ρ. Obviously, we may convert C_ρ and B_ρ into normed linear space by introducing the following ρ-norm

$$\|f\|_\rho = \sup_x \frac{|f(x)|}{\rho(x)}.$$

Also, let C_ρ^0 be the subspace of all functions $f \in C_\rho$ for which $\lim_{|x|\to\infty}(f(x)/\rho(x))$ exists finitely.

The properties of linear positive operators acting from C_ρ to B_ρ and the Korovkin type theorems for them have been studied by the first author who has established the following basic theorem, see [**3, 4**].

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