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# PIECEWISE WEIGHTED MEAN FUNCTIONS AND HISTOGRAMS

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ABSTRACT. Piecewise weighted mean functions are used to approximate histograms. For any histogram with uniform class intervals we determine piecewise interpolating weighted mean functions at least continuously differentiable in the approximation interval, preserving monotonicity and positivity of the given histogram, interpolating the frequencies at the middle point of each class interval, and satisfying a global area matching condition.

**1. Introduction.** Let $F = \{F_1, \dots, F_n\}$ be a histogram where $F_i$ is the frequency for the class interval $[X_i, X_{i+1}]$, with $X_{i+1} - X_i = h_i > 0$, $i = 1, \dots, n$. In order to smooth the histogram $F$, one can be interested in the construction of a function $s(x)$, at least continuously differentiable in $(X_1, X_{n+1})$, which satisfies the global area matching condition

$$\int_{X_1}^{X_{n+1}} s(x)\,dx = \sum_{i=1}^{n} h_i F_i, \tag{1.1}$$

or the conditions

$$\int_{X_i}^{X_{i+1}} s(x)\,dx = h_i F_i, \quad i = 1, \dots, n. \tag{1.2}$$

In addition, it is desirable that $s(x)$ reflects the shape of the histogram, which means that properties like monotonicity and/or positivity of $F$ should be preserved. In some recent papers histograms are approximated by using splines satisfying the area matching conditions (1.2). An algorithm which leads to a sufficient condition for the existence of positive and piecewise monotone quadratic splines as well as to their construction is derived in [**5**]. Necessary and sufficient conditions,

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