ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 28, Number 4, Winter 1998

# ON EQUAL SUMS OF CUBES

AJAI CHOUDHRY

ABSTRACT. The complete solution in positive or negative rationals of the Diophantine equation $x^3 + y^3 = u^3 + v^3$ was found by Euler. However, the complete solution in integers of this equation and of the related equation $x^3+y^3+z^3 = t^3$ has not been found earlier. This paper gives a complete solution of these equations in positive or negative integers as well as a complete solution in positive integers only.

**Introduction.** The complete solution of the Diophantine equation

$$x^3 + y^3 = u^3 + v^3 \tag{1}$$

in rational numbers, whether positive or negative, was first given by Euler. Writing $z, t$ for $-u, v$ respectively in (1), we get the related equation

$$x^3 + y^3 + z^3 = t^3. \tag{2}$$

Euler's solution has been presented by Hardy and Wright [**1**, pp. 199–200] who have stated that these equations give rise to a number of different problems, since we may look for solutions in (a) integers or (b) rationals and we may or may not be interested in the signs of solutions. They have further indicated that the complete solution of these equations in integers is not known. Hua Loo Keng [**2**, p. 290], while giving the complete rational solution of the equations, has also stated that, "The solutions to the equation $x^3+y^3+z^3+w^3 = 0$ present a very interesting problem. Unfortunately we still have not obtained a formula for all the solutions."

When we are not interested in the signs of the solutions, both equations (1) and (2) are equivalent to the equation

$$x_1^3 + x_2^3 + x_3^3 + x_4^3 = 0. \tag{3}$$

Received by the editors on January 24, 1997.