ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 29, Number 1, Spring 1999

QUALITATIVE PROPERTIES OF FUNCTIONAL DIFFERENTIAL EQUATIONS WITH "MAXIMA"

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ABSTRACT. In this paper some qualitative properties of the solutions for the functional differential equations with "maxima" of the form

$$[x(t) - p(t)x(t-\tau)]' + q(t) \max_{[t-\sigma,t]} x(s) = 0$$

are established.

1. Introduction. Consider the neutral differential equation

$$(1) \qquad [x(t) - p(t)x(t-\tau)]' + q(t) \max_{[t-\sigma,t]} x(s) = 0,$$

where $\tau > 0$, $\sigma \geq 0$ and $p, q \in C([t_0, \infty), R)$. The differential equations with "maxima" are often met in the applications, for instance, in the theory of automatic control [**8, 9**]. The qualitative theory of these equations has been developed relatively little. The existence of periodic solutions of the equations with "maxima" is considered in [**10**] and [**11**]. The oscillatory properties of Equation (1) are considered in [**1**–**3**]. The main goal of this paper is to discuss more comprehensively the oscillation and nonoscillation of Equation (1).

By a solution of (1) we mean a function x which is defined for $t \geq -\max(\sigma, \tau)$ and which satisfies (1) for $t \geq 0$. By the method of steps, we know that, for a given initial function $\phi \in C([-\max(\sigma,\tau), 0], R)$, there exists a unique solution defined for $t \geq -\max(\sigma,\tau)$ and which satisfies the initial condition for $-\max(\sigma,\tau) \leq t \leq 0$.

A nontrivial solution of (1) is said to be oscillatory if it is neither eventually positive nor eventually negative. Otherwise, the solution

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Received by the editors on August 26, 1996, and in revised form on May 1, 1997.
Key words and phrases. Functional differential equation with "maxima," oscillation, nonoscillation, asymptotic behavior.
AMS (1991) *Mathematics Subject Classification.* 34K15.
This work is supported by NNSF of China.