

**CYCLIC SUBGROUP SEPARABILITY
OF CERTAIN HNN EXTENSIONS OF
FINITELY GENERATED ABELIAN GROUPS**

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ABSTRACT. In this note we give characterizations for certain HNN extensions of finitely generated abelian groups to be cyclic subgroup separable.

1. Introduction. The residual finiteness of the Baumslag-Solitar groups $G_{k,l} = \langle t, a; t^{-1}a^k t = a^l \rangle$ were exhaustively studied and completely characterized by Baumslag and Solitar [5], Meskin [8] and Collins and Levin [7]. Their results can be summarized as follows:

Theorem A. *Let $G_{k,l} = \langle t, a; t^{-1}a^k t = a^l \rangle$. Then $G_{k,l}$ is residually finite if and only if $|k| = 1$ or $|l| = 1$ or $|k| = |l|$.*

Observing that the Baumslag-Solitar groups are HNN extensions with base group an infinite cyclic group, Andreadakis, Raptis and Varsos in a series of papers [2, 3, 4, 9] gave characterizations for the HNN extensions $G = \langle t, K; t^{-1}At = B, \varphi \rangle$ where the base group K is a finitely generated abelian group, to be residually finite. Motivated by the above results, Wong in [11] and [12] gave characterizations for the Baumslag-Solitar groups and some of the above HNN extensions to be subgroup separable.

In this note we shall extend the results of Andreadakis, Raptis and Varsos by giving characterizations for those HNN extensions to be cyclic subgroup separable.

Our main results are contained in Theorems 1–5. In addition, we shall give some applications.

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