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THE ERGODIC HILBERT TRANSFORM ON THE WEIGHTED SPACES $\mathfrak{L}_p(G, w)$

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ABSTRACT. We extend a theorem of Hunt, Muckenhoupt and Wheeden on weighted norm inequalities for the Hilbert transform. Our generalization to locally compact abelian groups is formulated in terms of the ergodic Hilbert transform and the ergodic A_p-condition.

1. Introduction. We consider the question of the continuity of the ergodic Hilbert transform from a weighted $\mathfrak{L}_p$-space $\mathfrak{L}_p(G, w)$ into itself, where G is a locally compact abelian group. The classical result for $G = \mathbf{R}$ or $\mathbf{T}$ is that the A_p-condition for a weight w characterizes the continuity of the Hilbert transform. This result was given by Hunt, Muckenhoupt and Wheeden [**6**], which we state in the following theorem.

Theorem 1.1. *Let $G = \mathbf{R}$ or $\mathbf{T}$, let $T = H$ or MH, the Hilbert transform or maximal Hilbert transform, and let w be a nonnegative function in $\mathfrak{L}^1_{\text{loc}}(G)$. If $1 < p < \infty$, then the weighted norm inequality*

$$\int_G |Tf(t)|^p w(t)\,dt \le K_p \int_G |f(t)|^p w(t)\,dt \tag{1.1}$$

holds for every $f \in \mathfrak{L}_p(G, w)$ if and only if the weight w satisfies the A_p-condition

$$\sup_I \frac{1}{|I|}\int_I w(t)\,dt \left(\frac{1}{|I|}\int_I w^{-1/(p-1)}(t)\,dt\right)^{p-1} \le A_p. \tag{A_p}$$

When G is any locally compact abelian group, the ergodic A_p-condition, defined in terms of a continuous homomorphism from $\mathbf{R}$ into

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