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GROUPS OF ISOMETRIES OF A TREE AND THE CCR PROPERTY

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1. Introduction. Let X be a homogeneous tree of order $q+1 \geq 3$. Let Ω be the tree boundary. Let $\mathrm{Aut}\,(X)$ be the locally compact group of all isometries of X. The reader is referred to [**10**] or [**3**] for undefined notions and terminology. In [**5**] a locally compact group G is called a CCR-group if $\pi(f)$ is a compact operator for every $f \in L^1(G)$ and for every $\pi \in \hat{G}$ where $\hat{G}$ is the set of equivalence classes of all unitary continuous irreducible representations of G. Every CCR-group is a type I group [**2**]. $\mathrm{Aut}\,(X)$ is a CCR-group, see [**7**] or [**3**, p. 113]. Also, $PGL(2,\mathbf{Q}_p)$ where $\mathbf{Q}_p$ is the field of the p-adic numbers, is a CCR-group [**9**]. It is known that $PGL(2,\mathbf{Q}_p)$ may be realized as a closed subgroup of $\mathrm{Aut}\,(X)$, for some tree X,in such a way that $PGL(2,\mathbf{Q}_p)$ acts transitively on X and Ω. If G is a locally compact totally disconnected group, then the property CCR is equivalent to the fact that every unitary irreducible representation of G is admissible, see Section 2 below. On the other hand, in the present paper, we prove that if G is a closed unimodular CCR-subgroup of $\mathrm{Aut}\,(X)$ acting transitively on X, then G acts transitively on Ω. We conjecture that the converse is true. This conjecture is supported by the fact that all noncuspidal irreducible representations of a closed subgroup of $\mathrm{Aut}\,(X)$ acting transitively on X and on Ω are in fact admissible representations. This follows from the classification given in [**3**, p. 84]. It is also true that every irreducible subrepresentation of the regular representation is admissible [**4**, p. 6].

2. The result. There exists a K-invariant probability measure on the tree boundary, Ω, say ν. Let $P(g,\omega)$ be the Poisson kernel associated with ν, that is, $P(g,\omega) = (d\nu_g/d\nu)(\omega)$ for $g \in \mathrm{Aut}\,(X)$ and $\omega \in \Omega$ with $\nu_g(\omega) = \nu(g^{-1}\omega)$, see [**3**, pp. 34–35]. For every $t \in \mathbf{R}$, we

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