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# AN EXPLICIT UPPER BOUND FOR THE RIEMANN ZETA-FUNCTION NEAR THE LINE $\sigma = 1$

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ABSTRACT. In this paper we give the following explicit estimate for the Riemann zeta-function. Let $t \geq 2$. For $1/2 \leq \sigma \leq 1$,

$$|\zeta(\sigma + it)| \leq 175 t^{46(1-\sigma)^{3/2}} \log^{2/3} t;$$

for $\sigma \geq 1$,

$$|\zeta(\sigma + it)| \leq 175 \log^{2/3} t.$$

**1. Introduction.** In regard to the prime number theorem, the zero-free region of the Riemann zeta-function plays an important role. The best known zero-free region of the Riemann zeta-function asserts that there are no zeros of $\zeta(\sigma + it)$ for $\sigma > 1 - c(\log t)^{-2/3}(\log\log t)^{-1/3}$ and $t \geq t_0$ where $c$ and $t_0$ are absolute positive constants. This zero-free region was established by using the methods from Vinogradov and Korobov, and the principal tool was an upper bound for the Riemann zeta-function near the line $\sigma = 1$.

In 1963, Richert proved the following result, see [**11**]. For $1/2 \leq \sigma \leq 1$ and $t \geq 2$, there exists an absolute constant $A$ such that

$$(*) \qquad |\zeta(\sigma + it)| \leq A t^{B(1-\sigma)^{3/2}} \log^{2/3} t,$$

with $B = 100$. In 1975, Ellison proved the same result as $(*)$ with $B = 86$ and $A = 2100$, see [**7**]. There are also other results with sharpened numbers $B$, see [**10, 1**].

In some applications one needs to have the result completely explicit. That is, to have the number $A$ calculated out. To prove the above result with a reasonable size of the number $A$, it is convenient to use

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