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# ON HANKEL CONVOLUTION EQUATIONS IN DISTRIBUTION SPACES

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ABSTRACT. In this paper we study Hankel convolution equations in distribution spaces. Solvability conditions for Hankel convolution equations are obtained. Also we investigated hypoelliptic Hankel convolution equations.

**1. Introduction.** The Hankel integral transformation is usually defined by

$$h_\mu(\phi)(x) = \int_0^\infty (xt)^{1/2} J_\mu(xt)\phi(t)\,dt, \quad x \in I = (0,\infty),$$

where $J_\mu$ denotes the Bessel function of the first kind and order $\mu$. Throughout this paper $\mu$ always will be greater than $-1/2$, and we will denote by $I$ the real interval $(0,\infty)$.

Zemanian [**25, 26** and **27**] investigated the $h_\mu$ transformation on generalized function spaces. He introduced in [**25**] the space $\mathcal{H}_\mu$ constituted by all those complex valued and smooth functions $\phi$ defined on $I$ such that, for every $m, k \in \mathbf{N}$,

$$\gamma^\mu_{m,k}(\phi) = \sup_{x\in(0,\infty)} \left| x^m \left(\frac{1}{x}D\right)^k [x^{-\mu-1/2}\phi(x)]\right| < \infty.$$

The space $\mathcal{H}_\mu$ is Fréchet when it is endowed with the topology generated by the family $\{\gamma^\mu_{m,k}\}_{m,k\in\mathbf{N}}$ of seminorms. It was established, [**25**, Lemma 8] that $h_\mu$ is an automorphism of $\mathcal{H}_\mu$. The Hankel transformation is defined on $\mathcal{H}'_\mu$, the dual space of $\mathcal{H}_\mu$, as the adjoint of the $h_\mu$-transformation of $\mathcal{H}_\mu$, and it is denoted by $h'_\mu$. More recently,

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