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SURFACES IN $\mathbf{P}^5$ WHICH DO NOT ADMIT TRISECANTS

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ABSTRACT. We study the trisecant lines of surfaces embedded in $\mathbf{P}^5$. We are mainly interested in surfaces defined over the algebraic closure of a finite field, embedded in the Grassmannian $G(1,3)$ of lines of $\mathbf{P}^3$ and having no trisecant line.

1. Introduction. The main actors in this paper are surfaces embedded in $G(1,3)$, the Grassmannian of lines of $\mathbf{P}^3$ over any algebraically closed field $\mathbf{K}$ and in particular we are interested in the case in which $\mathbf{K}$ is the algebraic closure of a finite field $GF(q)$, $q = p^h$, $h \geq 1$, p prime. We will study surfaces X contained in $G(1,3)$ which have the particularity to contain no trisecant lines, in the sense of Definition 2.1, and as we will see these are very few. Actually, since our tools are algebraic geometric, most of our results hold for a surface in $\mathbf{P}^5$, not just for a surface in $G(1,3)$ seen as a smooth quadric hypersurface of $\mathbf{P}^5$, see Theorems 4.1 and 4.2. Denote by $PG(n,q)$ the projective space of dimension n over $GF(q)$. There is a close relation between such surfaces and objects coming from Galois geometries, namely, K-caps in $PG(n,q)$.

A K-*cap* in $PG(n,q)$ is a set of K points, no three of which are collinear, cf. [**11**, p. 285]. A K-cap of $PG(2,q)$ is also called a K-*arc*. The maximum value of K for which there exists a K-cap in $PG(n,q)$ is denoted by $m_2(n,q)$, cf. [**11**, p. 285]. This number $m_2(n,q)$ is only known, for arbitrary q, when $n \in \{2,3\}$. With respect to the other values of $m_2(n,q)$, only upper bounds are known. Constructing a K-cap of size $m_2(n,q)$, $n \geq 4$ seems to be an extremely hard problem.

Some authors looked at caps contained in algebraic varieties such as quadrics or Hermitian varieties. Here we are substantially interested in

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