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EMBEDDING DERIVATIVES OF $\mathcal{M}$-HARMONIC FUNCTIONS INTO L^p SPACES

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ABSTRACT. A characterization is given of those Borel measures μ on B, the unit ball in C^n, such that differentiation of order m maps the $\mathcal{M}$-harmonic Hardy space $\mathcal{H}^p$ boundedly into $L^q(\mu)$, $0 < q < p < +\infty$.

1. Introduction. Let B denote the unit ball in C^n, $n \geq 1$, and m the $2n$-dimensional Lebesgue measure on B normalized so that $m(B) = 1$, while σ is the normalized surface measure on its boundary S. We set $d\tau(z) = (1-|z|^2)^{-1-n}\,dm(z)$. For the most part, we will follow the notation and terminology of Rudin [**10**]. If $\alpha > 0$ and $\xi \in S$, the corresponding Koranyi approach region is defined by

$$D_\alpha(\xi) = \{z = r\eta \in B : |1 - \langle \eta, \xi\rangle| < \alpha(1-r)\},$$

those regions are equivalent to the standard approach regions $\{z \in B : |1-\langle z, \xi\rangle| < 2^{-1}\beta(1-|z|^2),\ \beta > 1\}$. For any function u on B we define a scale of maximal functions by

$$M_\alpha u(\xi) = \sup\{|u(z)| : z \in D_\alpha(\xi)\}.$$

Let $\tilde{\Delta}$ be the invariant Laplacian on B. That is,

$$(\tilde{\Delta} u)(z) = \frac{1}{n+1}\Delta(u \circ \phi_z)(0), \quad u \in C^2(B),$$

where Δ is the ordinary Laplacian and ϕ_z the standard involutive automorphism of B taking 0 to z, see [**10**]. A function u defined on B is $\mathcal{M}$-harmonic, $u \in \mathcal{M}$, if $\tilde{\Delta} u = 0$.

For $0 < p < \infty$, $\mathcal{M}$-harmonic Hardy space $\mathcal{H}^p$ is defined to be the space of all functions $u \in \mathcal{M}$ such that $M_\alpha u \in L^p(\sigma)$ for some $\alpha > 0$, $\|u\|_p = \|M_\alpha u\|_p$. This definition is independent of α and the

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