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QUOTIENT NEARRINGS OF SEMILINEAR NEARRINGS

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1. Introduction. All nearrings in this paper will be right nearrings. Let R^n denote the n-dimensional Euclidean group. In [**1**], we showed that if λ is a continuous map from R^n to R and a multiplication $*$ is defined on R^n by $v * w = \lambda(w)v$, then $(R^n, +, *)$ is a topological nearring if and only if $\lambda(av) = a\lambda(v)$ for all $v \in R^n$ and $a \in \operatorname{Ran}(\lambda)$ where $\operatorname{Ran}(\lambda)$ denotes the range of λ. Any map from R^n to R with this property will be referred to as a *semilinear map*. Such maps are quite abundant. For example, let P be any homogeneous polynomial of degree m. That is, $P(tv_1, tv_2, \dots, tv_n) = t^m P(v_1, v_2, \dots, v_n)$ for all $t \in R$ and all $v \in R^n$. Define $\lambda(v) = |P(v)|^{1/m}$. Then λ is a semilinear map. If m is odd, one can also obtain a semilinear map λ by defining $\lambda(v) = (P(v))^{1/n}$. By a *semilinear nearring*, we mean a topological nearring $(R^n, +, *)$ where the multiplication $*$ is induced by a semilinear map λ, and we will denote such a nearring by $N_\lambda(R^n)$. In [**1**], we determined all the ideals (here, ideal means two-sided ideal) of a semilinear nearring. In this paper we show that every nonzero quotient nearring of a semilinear nearring is isomorphic to a semilinear nearring, and we determine precisely when two quotient nearrings of $N_\lambda(R^n)$ are isomorphic. Among other things, we will see that, although $N_\lambda(R^n)$ may have infinitely many quotient nearrings, it has, up to isomorphism, only finitely many and, in fact, this number cannot exceed $n+1$.

2. The results. Let $N_\lambda(R^n)$ be a semilinear nearring, and let

(2.1)
$$C(\lambda) = \{w \in R^n : \lambda(v + aw) = \lambda(v) \text{ for all } a \in R \text{ and all } v \in R^n\}.$$

In [**1**], we proved the following

Theorem 2.1. *Let λ be any nonconstant semilinear map from R^n to R. Then $C(\lambda) \subseteq \lambda^{-1}(0)$, $C(\lambda)$ is a linear subspace of R^n and the*

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