ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 29, Number 2, Summer 1999

SOURCE-TYPE SOLUTIONS TO POROUS MEDIUM EQUATIONS WITH CONVECTION II

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ABSTRACT. Existence and uniqueness of nonnegative source-type solutions to porous medium equations with convection are investigated in one space dimension. We consider the case when both diffusion and convection nonlinearities are pure powers, namely, r^m, $m > 1$, and r^q, $q \in (1, m)$, respectively. Results on the behavior of the source-type solution for small times and estimates of its support are also provided.

1. Introduction. We investigate existence and uniqueness of nonnegative source-type solutions to degenerate convection-diffusion equations in one space dimension. More precisely, we consider the following problem:

$$u_t + (u^q)_x - (u^m)_{xx} = 0 \quad \text{in} \quad \mathbf{R} \times (0, +\infty), \tag{1.1}$$

with initial data

$$u(0) = M\delta, \tag{1.2}$$

where M is a positive real number, δ denotes the Dirac mass at $x = 0$, and q and m are nonnegative real numbers satisfying

$$m > 1, \quad q \in (1, m). \tag{1.3}$$

This paper is a continuation of [**10**], where existence and uniqueness of the nonnegative source-type solution to (1.1)–(1.2) are proved for $m > 1$ and $q \geq m$, while the case $m = 1$ and $q > 1$ is completely solved in [**3**]. These investigations were motivated by the study of the long-time behavior of the solutions to (1.1) with nonnegative and integrable initial data u_0 satisfying $|u_0|_{L^1} = M$. Indeed, when $q = m+1$, the long-time profile of these solutions is described by the source-type solution

Received by the editors on October 3, 1996, and in revised form on June 27, 1997.
1991 AMS *Mathematics Subject Classification.* 35K65.