ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 29, Number 2, Summer 1999

SYMMETRY INCREASING BIFURCATIONS VIA COLLISIONS OF ATTRACTORS

CORDULA HEINRICH

ABSTRACT. In 1988, Chossat and Golubitsky observed numerically, in discrete dynamical systems equivariant under the action of a finite group, a phenomenon for which they coined the name *symmetry increasing bifurcation*. They observed that, while varying a parameter, conjugate attractors of such a system may collide yielding an attractor with larger symmetry group than before.

One of the questions arising in this context is the following: Given a group Γ, which subgroups Σ of Γ are admissible in the sense that Σ-symmetric attractors of a Γ-equivariant mapping may undergo a symmetry increasing bifurcation?

In this paper we extend the approach to solve this problem made by Dellnitz and Heinrich. We construct collisions of attractors at arbitrary reflection hyperplanes, as well as collisions which take place at points of trivial isotropy. Combining these results we are able to give necessary and sufficient criterions for admissibility of these collisions.

1. Introduction. Discrete dynamical systems on $\mathbf{R}^n$ equivariant under the action of a finite group Γ typically possess attractors displaying symmetry. More precisely, these attractors as a set are invariant under the action of a subgroup of Γ. If a parameter is introduced into the system, preserving the symmetry, then one can often observe that these attractors collide with conjugate attractors yielding an attractor with larger symmetry than before. This phenomenon was observed by Grebogi, Ott, Romeiras and Yorke [**11**] and Chossat and Golubitsky [**6**], who named these transitions *symmetry increasing bifurcations*.

Since then, other mechanisms by which symmetry can be increased have been observed. In particular, three mechanisms have been described in [**7**]. Apart from collisions, also "explosions" of attractors may take place, see also King and Stewart [**13**], and, for continuous groups, an attractor may start to "drift" along its group orbit yielding larger symmetry than before, see Dellnitz, Golubitsky and Melbourne

Received by the editors on January 20, 1997, and in revised form on August 11, 1997.