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MULTIPOINT PADÉ-TYPE APPROXIMATION: AN ALGEBRAIC APPROACH

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ABSTRACT. In this paper multipoint Padé-type approximants are formally introduced by defining a linear functional on the space of certain rational functions with prescribed poles. Some expressions for the numerator, a compact formula and some error formulas are also given.

1. Introduction. In [**1**], Brezinski introduced the so-called Padé-type approximants in one point, giving an algebraic development for such a rational approximation. The basis of this approach consisted in defining a linear functional on the space of usual polynomials. Following the same procedure, Draux [**3, 4**] and González-Vera [**5**] constructed two-point (zero and infinity) Padé-type approximants, starting from a linear functional defined on the space of Laurent-polynomials. In [**12**], see also [**13**], Van Iseghem extended the former cases to the multipoint one following a different technique, namely, the Hermite polynomial interpolation.

In this paper we introduce multipoint Padé-type approximation in a similar way to Brezinski, using an idea employed by Njåstad in [**9**] to construct multipoint Padé approximants. We shall define a linear functional acting on the space of R-functions [**8**], when none of the points where the approximant is to be built on is infinity, or on the space of generalized R-functions (GR-functions), [**10**], otherwise. Since R-functions can be considered as a particular case of GR-functions, we shall restrict ourselves to this general case.

In Section 2 we shall define multipoint Padé-type approximants (MPTA) and introduce the notation to be used in the paper, recalling also the definition of GR-functions. Section 3 is concerned with the construction of MPTAs, of type $(k-1,k)$, i.e., the numerator is of degree $k-1$ and the denominator of degree k at most, and using

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