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THE DIOPHANTINE EQUATION $ax^5 + by^5 + cz^5 = au^5 + bv^5 + cw^5$

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ABSTRACT. A parametric solution of the Diophantine equation $ax^5 + by^5 + cz^5 = au^5 + bv^5 + cw^5$ is obtained when a, b and c are distinct nonzero integers such that $a+b+c=0$.

While several parametric solutions of the Diophantine equation

$$x^5 + y^5 + z^5 = u^5 + v^5 + w^5$$

are known [**1, 2, 3, 4, 5**], the Diophantine equation

$$ax^5 + by^5 + cz^5 = au^5 + bv^5 + cw^5 \tag{1}$$

has not been considered earlier. In this paper we give a parametric solution of (1) when a, b, c are integers such that

$$a + b + c = 0. \tag{2}$$

The solution reduces to a trivial one when $abc(a-b)(b-c)(c-a) = 0$. Thus, nontrivial solutions of (1) are obtained only when a, b, c are distinct nonzero integers satisfying the relation (2).

To solve (1), we write

$$\begin{aligned} x &= p\theta + \alpha, \qquad y = q\theta + \alpha, \qquad z = r\theta + \alpha, \\ u &= -b\theta + \beta, \qquad v = a\theta + \beta, \qquad w = \beta \end{aligned} \tag{3}$$

where p, q, r, α and β are arbitrary, and we will take $\alpha\beta \neq 0$. Substituting these values in (1), we get the following fifth degree equation in θ:

$$\begin{aligned} a\{(p\theta+\alpha)^5 - (-b\theta+\beta)^5\} + b\{(q\theta+\alpha)^5 - (a\theta+\beta)^5\} \\ + c\{(r\theta+\alpha)^5 - \beta^5\} = 0. \end{aligned} \tag{4}$$

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