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ASYMPTOTIC THEORY FOR A GENERAL THIRD-ORDER DIFFERENTIAL EQUATION OF EULER TYPE

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1. Introduction. In this paper we investigate the asymptotic form of three linearly independent solutions of the third-order differential equation

$$\{q(x)(q(x)y'(x))'\}' + \{(q_1(x)y(x))' + q_1(x)y'(x)\}/2 + (p_0(x)y'(x))' + p_1(x)y(x) = 0 \tag{1.1}$$

as $x \to \infty$. The functions q, q_1, p_0 and p_1 are defined on the interval $[a, \infty)$ with q nowhere zero. We do not need to restrict ourselves to real-valued coefficients nor to powers of x. Our aims are to identify relations between q, q_1, p_0 and p_1 corresponding to an Euler case for (1.1) and to obtain the asymptotic forms of the solutions in these cases. The various conditions imposed on the coefficients will be introduced when they are required in the development of the method. Al-Hammadi [**2**] considers (1.1) in the case where the solutions all have a similar exponential factor. A third-order equation similar to (1.1) has been considered previously by Al-Hammadi [**1**], Unsworth [**7**] and Pfeiffer [**6**].

Eastham [**4**] considered an Euler case for a fourth-order differential equation and showed that this case represents a borderline between situations where all solutions have a certain exponential character as $x \to \infty$ and where only two solutions have this character. The Euler cases for (1.1) we referred to, are given by

Case A.

$$\frac{q_1'}{q_1} \sim \text{const.} \times \frac{p_0}{q^2}, \tag{1.2}$$

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