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SPLITTING OF LINEAR SYSTEMS WITH IMPULSES

RAÚL NAULIN AND MANUEL PINTO

ABSTRACT. In this paper we study some dichotomic properties of the impulsive system $y' = [A(t) + B(t)]y$, $\Delta y(t_k) = [C_k + D_k]y(t_k)$. We prove that if the nonperturbed system $x' = A(t)x$, $\Delta x(t_k) = C_k x(t_k)$ has an exponential dichotomy with projection P and $PA(t) = A(t)P$, $PC_k = C_k P$, it is satisfied for all values of t and k, then there exists a change of variables $y(t) = S(t)z(t)$, reducing the perturbed system to the form $z' = [A(t) + \tilde{B}(t)]z$, $\Delta z(t_k) = [C_k + \tilde{D}_k]z(t_k)$, with the properties $P\tilde{B}(t) = \tilde{B}(t)P$, $P\tilde{D}_k = \tilde{D}_k P$. From this result follows a theorem of roughness for exponential dichotomies of impulsive systems.

1. Introduction. In the following J will denote the interval $[t_0, \infty)$; V^n will stand for the space R^n or C^n; for a vector $x \in V^n$, $|x|$ will be some fixed norm in V^n; for an $n \times n$ matrix A, $|A|$ will denote the corresponding matrix norm. In this paper the symbol $\{t_k\}$ identifies a strictly increasing sequence, contained in (t_0, ∞), with the property $\lim_{k\to\infty} t_k = \infty$. The interval $(t_{k-1}, t_k]$ will be denoted by J_k. Finally, let us denote $N = \{1, 2, 3, \dots\}$.

The theory of equations with impulsive effect is a recent branch of the theory of differential equations. Beginning with the work of Mil'man and Myshkis [**11**], this theory has been developed by the contribution of many researchers: Halanay and Wexler [**8**], Bainov et al. [**2**], Lakshmikantham et al. [**9**], etc. The theoretical questions which arise in this area attract the attention of analysts and applied mathematicians because of an increasing number of their applications to semiconductor theory, quantum mechanics, ecology, biomathematics and control theory [**2, 8, 9**].

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