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ON THE ELEMENTARY PROOF OF THE PRIME NUMBER THEOREM WITH A REMAINDER TERM

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ABSTRACT. In this paper we have improve the remainder term of the prime number theorem by using the elementary method, Selberg's method, and obtained

$$\pi(x) = \operatorname{li} x + O\{x \exp(-\log^{(1/2)-\varepsilon} x)\},$$

where $\varepsilon > 0$ is an arbitrarily small constant, and the O is dependent on ε.

1. Introduction. Let $\pi(x)$ denote the number of prime numbers not exceeding x. The elementary proof of the prime number theorem was obtained by Selberg in 1949 and his method was modified by several mathematicians. In 1962 and 1964, Bombieri [**2**] and Wirsing [**6**] respectively and independently proved

$$\pi(x) = \operatorname{li} x + O\bigg(\frac{x}{\log^A x}\bigg),$$

where A is an arbitrary positive constant. In 1970, Diamond and Steinig [**3**] proved

$$\pi(x) = \operatorname{li} x + O\{x \exp(-(\log x)^{1/7} (\log\log x)^{-2})\}.$$

In 1973, А.Ф. Лаврик and А.Ш. Собиров [**5**] proved

$$\pi(x) = \operatorname{li} x + O\{x \exp(-(\log x)^{1/6} (\log\log x)^{-3})\}.$$

In this paper we have modified Selberg's method again and obtained

$$\pi(x) = \operatorname{li} x + O(x \exp(-\log^{(1/2)-\varepsilon} x)),$$

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