

INVARIANT HYPERBOLIC TORI FOR  
HAMILTONIAN SYSTEMS WITH RÜSSMANN  
NONDEGENERACY CONDITIONS

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ABSTRACT. Following the procedure designed by Graff for proving the persistence theorem of invariant hyperbolic tori for Hamiltonian systems with some modifications, we obtain in this paper a KAM theorem for Hamiltonian systems with hyperbolic fixed point. Because the Hamiltonian of unperturbed systems satisfies the Rüssmann nondegeneracy condition, this generalizes the well-known result of Graff.

**1. Introduction and main results.** In the classical KAM theory, a stronger nondegeneracy condition is required, see [1, 7, 8]. To weaken that condition is, currently, an attractive topic, and there have been some profound works for Hamiltonian systems, for example, [2–4, 11, 15, 17]. However, in our opinion, the real weakening of the degeneracy condition should look like

“image of the frequency map  $y \rightarrow w(y)$  does not lie in a hyperplane of the frequency space.”

This is precisely Rüssmann nondegeneracy condition [13]. Pöschel’s work [9, 10] may imply Rüssmann’s conjecture, because Rüssmann’s condition actually restricts the frequency vectors of the unperturbed system to some “twisted manifold.” Recently, Rüssmann’s conjecture was proved in [16]. Almost certainly this can be done for other KAM-type theorems.

In the present paper, we shall consider such degeneracy problems. More precisely, we shall prove the following result about the existence of invariant hyperbolic tori for Hamiltonian systems with Rüssmann nondegeneracy.

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