ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 29, Number 3, Fall 1999

ON THE SECOND HILBERT 2-CLASS FIELD OF REAL QUADRATIC NUMBER FIELDS WITH 2-CLASS GROUP ISOMORPHIC TO $(2, 2^n)$, $n \geq 2$

ELLIOT BENJAMIN

ABSTRACT. Let k be a real quadratic number field with $C_{k,2}$, the 2-Sylow subgroup of its ideal class group, isomorphic to $\mathbf{Z}/2\mathbf{Z} \times \mathbf{Z}/2^n\mathbf{Z}$, $n \geq 2$, such that $\mathrm{Gal}\,(k_2/k)$, the galois group over k of the second Hilbert 2-class field of k, is nonabelian. We describe conditions for which we can further refine $\mathrm{Gal}\,(k_2/k)$ in terms of its group structure being modular, metacyclic-nonmodular, or nonmetacyclic, when a prime congruent to 3 mod 4 does not divide the discriminant of k.

1. Preliminaries. Let k be a real quadratic number field with $C_{k,2}$, the 2-Sylow subgroup of its ideal class group, isomorphic to $\mathbf{Z}/2\mathbf{Z} \times \mathbf{Z}/2^n\mathbf{Z}$, $n \geq 2$, which we will denote by $(2, 2^n)$. We let k_1 denote the Hilbert 2-class field of k, i.e., the maximal unramified (including the infinite primes) abelian field extension of k which has degree a power of 2. Then $C_{k,2} \cong \mathrm{Gal}\,(k_1/k)$, the galois group of k_1 over k, and we let $k_2 = (k_1)_1$. In our earlier work we have completely determined when $\mathrm{Gal}\,(k_2/k)$ is abelian, [**1, 2**]. In certain cases, particularly when a prime congruent to 3 mod 4 divides d_k, the discriminant of k, we have utilized information about the capitulation of ideal classes in unramified quadratic extensions of k in order to further classify $\mathrm{Gal}\,(k_2/k$ in terms of its group structure being modular, metacyclic-nonmodular, or nonmetacyclic [**1, 2**]. In the present paper we extend the above classification for nonabelian $\mathrm{Gal}\,(k_2/k)$ to the remaining cases, i.e., when a prime congruent to 3 mod 4 does not divide the discriminant of k.

Recall that a group G is metacyclic if there exists a normal cyclic subgroup, N, of G such that G/N is also cyclic. Finite metacyclic 2-groups, G, for which $G/G' \cong (2, 2^n)$, $n \geq 2$, can be divided into two isomorphism types: the modular groups, $M_{n+2}(2)$, and those which

Received by the editors on April 9, 1996, and in revised form on August 19, 1997.