

**DETERMINANTS OF CERTAIN CLASSES
OF ZERO-ONE MATRICES WITH
EQUAL LINE SUMS**

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ABSTRACT. We study the possible determinant values of various classes of $n \times n$ zero-one matrices with fixed row and column sums. Some new results, open problems, and conjectures are presented.

1. Introduction. Let k, n be positive integers with $k \leq n$. Denote by $S(n, k)$ the set of zero-one $n \times n$ matrices with row sums and column sums equal to k .

There has been considerable interest in studying the determinant values of matrices in $S(n, k)$ and various subsets. This interest is motivated, among other things, by many interesting connections with graph theory and combinatorics (designs and configurations). So far the research in this area focused on the minimal positive value of determinants of matrices in $S(n, k)$, see, e.g., [7, 8, 11, 13], and on the maximal value of determinants for matrices in certain subsets of $S(n, k)$ and for certain values of n and k , see, e.g., [4, 6, 15], and see also the books [2, 16]. The main focus of the present paper is to describe in some cases the complete set of determinantal values of matrices in $S(n, k)$. We also consider the subset of symmetric matrices in $S(n, k)$ and the subset of $S(n, k)$ which is generated by powers of the standard circulant. Both subsets are of considerable interest in combinatorics.

Note that if $A \in S(n, k)$ with $\det(A) = t$, then one can interchange the first two rows of A to obtain a matrix in $S(n, k)$ with determinant

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