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STABILITY THEOREM FOR THE FEYNMAN INTEGRAL VIA TIME CONTINUATION

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ABSTRACT. Lapidus proved a stability theorem for the Feynman integral as a bounded linear operator on $L_2(\mathbf{R}^d)$ with respect to potential functions. We establish a stability theorem for the Feynman integral with respect to measures whose positive and negative variations are in the generalized Kato class. This is a partial extension of Lapidus's result. In fact, we develop our stability theorem under a more general setting in the sense that potential functions in Lapidus's paper are involved in the Kato class and the measures in this paper are involved in the generalized Kato class which generalizes substantially the Kato class.

0. Introduction. The purpose of this paper is to study the stability of the analytic (in time) operator-valued Feynman integral with respect to certain functions determined by measures in the generalized Kato class. In 1984, Johnson proved the dominated convergence theorem for the Feynman integral as an operator from $L_2(\mathbf{R}^d)$ to $L_2(\mathbf{R}^d)$ [**10**]. As far as we know, this is the first stability theorem for the Feynman integral. Since then, many mathematicians have proved stability theorems for the Feynman integral as either $\mathcal{L}(L_p(\mathbf{R}^d), L_{p'}(\mathbf{R}^d))$ theory [**6, 12**] or $\mathcal{L}(L_1(\mathbf{R}), C_0(\mathbf{R}))$ theory [**5**], where p is a real number such that $1 < p \leq 2$ and d is a positive integer such that $d < 2p/(2-p)$ for $1 < p < 2$.

In [**15**], Lapidus proved a stability theorem for the Feynman integral as an $\mathcal{L}(L_2(\mathbf{R}^d))$ theory with respect to the potential functions V, V_m, $m = 1, 2, \dots$, satisfying the following conditions: V_m converges to V almost everywhere in $\mathbf{R}^d$ and there exist $U \in L^1_{\text{loc}}$ and $W \in (L^p_{\text{loc}})_{\tilde{u}}$ such that, for all $m \geq 1$, $V_m^+ \leq U$ almost everywhere and $V_m^- \leq W$

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