

DUAL KRULL DIMENSION AND DUALITY

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Let R be a ring with Krull dimension. Then R has dual Krull dimension. Is there a relationship between the Krull dimension and the dual Krull dimension of R ? We shall show that in certain situations the dual Krull dimension is bounded above by the Krull dimension.

All rings are associative with nonzero identity and all modules are unital right modules. The Hopkins-Levitzki theorem asserts that every right Artinian ring is right Noetherian. In other words, every ring with Krull dimension 0 has dual Krull dimension 0. More generally, if R is a right Artinian ring and M an R -module with Krull dimension, then M is Noetherian, i.e., if R has Krull dimension 0. Then any non-zero R -module with Krull dimension has dual Krull dimension 0. It is natural to wonder if this situation is typical, and we are led to ask the following questions.

Question 1. If R is a ring with Krull dimension, is the dual Krull dimension of R bounded above by the Krull dimension?

Question 2. If R is a ring with Krull dimension and M is an R -module with Krull dimension, is the dual Krull dimension of M bounded above by the Krull dimension of R ?

As we shall see, these questions are equivalent, i.e., the dual Krull dimension is bounded above by the Krull dimension for *any* ring with Krull dimension if and only if the dual Krull dimension of M is bounded above by the Krull dimension of R for any ring R with Krull dimension and any R -module M with Krull dimension.

Although Questions 1 and 2 are left unanswered in this paper, we do have some information. If R is one of the following types of rings:

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