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ISOMORPHISM CLASSES OF UNIFORM GROUPS

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ABSTRACT. In this paper we count isomorphism classes of uniform groups within a fixed near-isomorphism class.

1. Preliminaries. An almost completely decomposable group X is an extension of a completely decomposable group R by a finite group X/R. If $\exp(X/R) = h$, denote $^- : R \to \overline{R} = h^{-1}R/R$, $x \mapsto \bar{x} = h^{-1}x + R$ the natural epimorphism. Furthermore, $^-$ denotes also the induced homomorphism $^- : \operatorname{Aut} R \to \operatorname{Aut} \overline{R}$, $\alpha \mapsto \bar{\alpha}$, which is well defined by $\bar{\alpha}(\bar{x}) := \overline{\alpha(x)}$. Recall, cf. [**6**], that

$$\operatorname{Typ}\operatorname{Aut}\overline{R} = \{\xi \in \operatorname{Aut}\overline{R} \mid \forall_{\tau \in T_{\mathrm{cr}}(R)} \xi\overline{R(\tau)} = \overline{R(\tau)}\}$$

is the set of *type automorphisms* of $\overline{R}$. Let $R = \bigoplus_{j=1}^{n} \langle x_j \rangle_*^R$, where $\mathbf{x} = (x_1, \dots, x_n)$ is an h-decomposition basis, i.e., $\operatorname{hgt}_p^R(x_j) \in \{0, \infty\}$ for all j and all primes p dividing h. Then $\bar{\mathbf{x}} = (\bar{x}_1, \dots, \bar{x}_n)$ is called an *induced decomposition basis* of $h^{-1}R/R$. We write $\mathbf{Z}_h := \mathbf{Z}/h\mathbf{Z}$. Let $\mathbf{a} = (a_1, \dots, a_r)$ be a basis of $X/R \subseteq h^{-1}R/R$. Then the basis elements a_i may be written as linear combinations of the induced decomposition basis $a_i = \sum_{j=1}^{n} \alpha_{ij}\bar{x}_j$, for $i = 1, \dots, r$, where $\alpha_{ij} \in \mathbf{Z}_h$. The $(r \times n)$-matrix $M = (\alpha_{ij})_{\substack{i=1,\dots,r \\ j=1,\dots,n}} \in \mathbf{M}^{r\times n}(\mathbf{Z}_h)$ is called *representing matrix* of X over R relative to $\mathbf{a}$ and $\bar{\mathbf{x}}$.

A group X is called *p-local* for a prime p if the regulator quotient $X/R(X)$ is a (finite) p-group.

Definition 1.1. Let p be a prime and e, n, r natural numbers. Let $T = (\tau_1, \dots, \tau_n)$ be an ordered n-tuple of pairwise incomparable types, where $\tau_i(p) \neq \infty$ each i. Then $\mathcal{C}(T, p, e, r)$ denotes the class of almost completely decomposable groups X such that

(1) $T = T_{\mathrm{cr}}(X)$ is the critical typeset of X,

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