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STRICTLY NONZERO CHARGES

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This paper is dedicated to the memory of our dear friend, Rae Michael Shortt.

Kelley in [**4**] discovered necessary and sufficient conditions on a Boolean algebra to admit a strictly positive bounded real-valued charge. As was noted in [**5**], the same conditions also characterize Boolean algebras that admit strictly nonzero bounded real-valued charges.

If G is a group and $\mathcal{A}$ is a Boolean algebra when would there exist a charge

$$\mu : \mathcal{A} \to G$$

which is strictly nonzero in the sense that $\mu(A) \neq 0$ whenever $A \in \mathcal{A}$ and $A \neq \emptyset$? The present paper is devoted to a study of this problem and its ramifications. We shall start with a result which says that to study group valued charges one has to look at only commutative groups. $\mathcal{A}, \mathcal{B}, \ldots$ stand for Boolean algebras or fields of sets and G stands for a group written additively.

Proposition 1. *If $\mu : \mathcal{A} \to G$ is a charge, there exists an abelian subgroup H of G such that $\mu(A) \in H$ whenever $A \in \mathcal{A}$.*

Proof. Let $D = \{\mu(A) : A \in \mathcal{A}\}$, the range of μ. If A and $B \in \mathcal{A}$, then $\mu(A)+\mu(B)-\mu(A\cap B) = \mu(A\cup B) = \mu(B\cup A) = \mu(B)+\mu(A)-\mu(A\cap B)$. Thus, if $x, y \in D$, then $x+y = y+x$. This implies that $\langle D\rangle$, the group generated by D, is commutative. □

Thus we assume that *all groups are abelian.* The next results says

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