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M-FREE ABELIAN GROUPS

MANFRED DUGAS, SHALOM FEIGELSTOCK AND JUTTA HAUSEN

ABSTRACT. We study M-free abelian groups with M-basis X, i.e., each map $f : X \to M$ extends uniquely to a homomorphism $\varphi : A \to M$. We will find conditions under which X generates a direct summand of A.

If F is an object in a concrete category, X a nonempty set and $i : X \to F$ a map, then F is free on the set X if for each M in the category and for each map $f : X \to M$ there is a morphism $\varphi : F \to M$ such that $\varphi \circ i = f$, cf. [**7**]. We will investigate, in the category of abelian groups only, which objects are "free" if, in the above definition, "each M" is replaced by "some fixed M." The answer, of course, depends on what kind of abelian group M actually is.

Definition. Let A, M be abelian groups and X a subset of A. Then A is *M-free with M-basis* X if, for each map $f : X \to M$, there is a *unique* $\varphi \in \operatorname{Hom}(A, M)$ such that $\varphi \restriction_X = f$ where $\varphi \restriction_X$ is the restriction of φ to X.

We say that A is *split-M-free* if $A = H \oplus \langle X \rangle$ such that $\langle X \rangle$ is free abelian with basis X and $\operatorname{Hom}(H, M) = 0$.

Of course, split-M-free implies M-free, and the main purpose of this paper is to investigate for which abelian groups M we have that all M-free groups A are actually split-M-free.

Let $\operatorname{Cent}(R)$ denote the center of a ring R. We will prove:

Main theorem. *Let A be M-free with M-basis X and M slender. If either*

(a) *M is countable and* $\operatorname{End}(M)^+$, *the additive group of the endomorphism ring of M, is free abelian, or*

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