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A GEOMETRIC SETTING FOR SOME PROPERTIES OF TORSION-FREE MODULES

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Dedicated to Jim Reid

1. Introduction. In this note we give examples of how certain properties of torsion-free modules that have been of recent interest arise in algebro-geometric settings. In particular, we find subrings of function fields whose torsion-free modules behave in ways similar to that of torsion-free abelian groups, and we indicate how these module-theoretic properties distinguish the associated rings and their geometry.

Some of our results are expository, the necessary commutative algebra having been developed elsewhere. In Section 2, relying on the work in [**7**], we interpret some Krull-Schmidt properties for modules geometrically. Our focus in this section is the Noetherian case, so it is natural that we find geometric analogues for the algebraic characterizations in [**7**].

In Section 3, we report on some recent results on the existence of finite character Prüfer domains in function fields. Here the "geometry" is more abstract in that our Prüfer rings do not arise as coordinate rings of varieties. Instead, they are intersections of valuation overrings of coordinate rings. The class of Noetherian Prüfer domains is precisely the class of Dedekind domains, so it is not surprising that one must search beyond coordinate rings of varieties to find Prüfer examples in function fields of transcendence degree greater than one.

We also push this construction a bit farther by indicating how h-local Prüfer domains can be found in function fields. An integral domain R is *h-local* if R has finite character and each nonzero prime ideal of R is contained in a unique maximal ideal of R. The classes of finite character and h-local Prüfer domains are playing an increasingly central role in a number of aspects of module theory, as is evidenced by the recent text [**5**].

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