

WEIGHTED COMPOSITION OPERATORS BETWEEN BLOCH-TYPE SPACES

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ABSTRACT. We discuss boundedness and compactness of composition operators followed by multiplication as operators between Bloch-type spaces of analytic functions on the unit disk.

1. Introduction. In this paper \mathbf{D} denotes the open unit disk, i.e., $\mathbf{D} = \{z \in \mathbf{C} : |z| < 1\}$. For $0 < \alpha < \infty$, let \mathcal{B}^α consist of all analytic functions f on \mathbf{D} satisfying the condition

$$\sup_{z \in \mathbf{D}} (1 - |z|^2)^\alpha |f'(z)| < \infty.$$

Note that $\mathcal{B}^1 = \mathcal{B}$, the usual Bloch space. For $f \in \mathcal{B}^\alpha$, define

$$\|f\|_{\mathcal{B}^\alpha} = |f(0)| + \sup_{z \in \mathbf{D}} (1 - |z|^2)^\alpha |f'(z)| < \infty.$$

With this norm \mathcal{B}^α is a Banach space.

For $0 < \alpha < 1$ and an analytic map $\varphi : \mathbf{D} \rightarrow \mathbf{D}$ the question when the composition operator C_φ given by

$$C_\varphi f = f \circ \varphi, \quad \text{for } f \in \mathcal{B}^\alpha,$$

is a bounded operator on \mathcal{B}^α was considered and solved first by Roan [8] and later by Madigan [5]. Boundedness and compactness of C_φ on the Bloch space $\mathcal{B} = \mathcal{B}^1$ were described by Madigan and Matheson [6].

The multipliers of the Bloch space $\mathcal{B} = \mathcal{B}^1$ were first characterized by Arazy [1]. For $\alpha \neq 1$, Zhu [12] characterized the multipliers of the spaces \mathcal{B}^α .

We will consider the question for which analytic functions u on \mathbf{D} and for which analytic mappings $\varphi : \mathbf{D} \rightarrow \mathbf{D}$ the weighted composition

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