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B-SCROLLS WITH NON-DIAGONALIZABLE SHAPE OPERATORS

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Dedicated to Professor D.E. Blair on the occasion of his sixtieth birthday

ABSTRACT. We study some Lorentzian surfaces in the three-dimensional Lorentzian space forms whose shape operators are not diagonalizable at least at one point. It is related to the so-called notion of 2-type surfaces. A local classification theorem in this respect is obtained.

1. Introduction. Let us denote by $\overline{M}_1^3(c)$ the standard model of a Lorentzian space form with constant curvature $c = 0, \pm 1$, that is, the Lorentz-Minkowski space L^3, the de Sitter space-time S_1^3 in E_1^4 and the anti de Sitter space-time H_1^3 in E_2^4, respectively. For $(n, \mu) = (3,1), (4,1)$ or (4,2), let E_μ^n be the corresponding pseudo-Euclidean space where $\overline{M}_1^3(c)$ is lying.

Suppose that $x : M_1^2 \to \overline{M}_1^3(c) \subset E_\mu^n$ is an isometric immersion of a two-dimensional connected Lorentzian surface into the three-dimensional Lorentzian space form. Denote by Δ the Laplacian operator of the Lorentzian surface M_1^2. The immersion x is said to be of finite type if each component of the position vector field of M_1^2 in E_μ^n, also denoted by x, can be written as a finite sum of eigenfunctions of the Laplacian operator Δ, that is, if

$$x = x_0 + x_1 + x_2 + \cdots + x_k, \tag{1.1}$$

where x_0 is a constant vector, $x_1, \dots, x_k$ are nonconstant maps satisfying $\Delta x_i = \lambda_i x_i$, $i = 1, \dots, k$. If, in particular, all eigenvalues

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