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THE HARNACK ESTIMATE FOR THE MODIFIED RICCI FLOW ON COMPLETE $\mathbf{R}^2$

SHU-CHENG CHANG, SHI-KUO HONG AND CHIN-TUNG WU

ABSTRACT. In this paper we prove the Harnack estimate for the evolved curvature R of the modified Ricci flow on complete $\mathbf{R}^2$.

1. Introduction. The Ricci flow is a second-order parabolic equation which deforms metric $g(t)$ in the direction of minus the Ricci curvature tensor $\operatorname{Ric}(g)$. That is, due to Hamilton ([**3**]), a family of Riemannian metrics $g(t)$, $t \in [0,T]$, is called a solution to the Ricci flow if

$$\frac{\partial}{\partial t} g(t) = -2 \operatorname{Ric}(g).$$

In particular, for a surface (Σ, g_0), the Ricci flow is given by

$$\begin{cases} \dfrac{\partial}{\partial t} g = -R \cdot g, \\ g(0) = g_0. \end{cases} \tag{1.1}$$

In [**4**], Hamilton studied the Ricci flow (1.1) on a compact Riemann surface S. He proved, among other results, that if the Riemann surface S is diffeomorphic to 2-sphere and the initial metric g_0 has positive curvature, that the solution of the normalized Ricci flow:

$$\begin{cases} \dfrac{\partial}{\partial t} g = (r - R) \cdot g, \\ g(0) = g_0, \end{cases}$$

converges to the limiting metric of positive constant curvature. Here r is the average value of the scalar curvature R. On the other hand, in [**7**], L.-F. Wu considered the Ricci flow (1.1) on a complete noncompact $\mathbf{R}^2$. She proved, if the covariant derivative of u_0 and the curvature of

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