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ORTHOGONAL LAURENT POLYNOMIALS AND QUADRATURE FORMULAS FOR UNBOUNDED INTERVALS: I. GAUSS-TYPE FORMULAS

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Dedicated to W.B. Jones on the occasion of his 70th birthday

ABSTRACT. We study the convergence of quadrature formulas for integrals over the positive real line with an arbitrary distribution function. The nodes of the quadrature formulas are the zeros of orthogonal Laurent polynomials with respect to the distribution function and with respect to a certain nesting. This ensures a maximal domain of validity and the quadratures are therefore called Gauss-type formulas. The class of functions for which convergence holds is characterized in terms of the moments of the distribution function. Moreover, error estimates are given when f satisfies certain continuity conditions. Finally, these results are applied to the family of distributions $d\varphi(x) = x^\alpha \exp\{-(x^{\gamma_1} + x^{-\gamma_2})\}\,dx$, $\gamma_1, \gamma_2 \geq 1/2$, $\alpha \in \mathbf{R}$.

1. Introduction. The main aim of this work is the approximate calculation of integrals of the form

$$I(f) = \int_0^\infty f(x)\,d\varphi(x),$$

φ being a distribution function on $\mathbf{R}^+$, i.e., a real-valued, bounded, nondecreasing function with infinitely many points of increase on any

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