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GLOBAL ATTRACTIVITY IN A GENOTYPE SELECTION MODEL

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ABSTRACT. We obtain a sufficient condition for global attractivity to occur in the delay difference equation

$$x_{n+1} = x_n \exp(\beta_n(1 - x_{n-\tau})/(1 + x_{n-\tau})).$$

This leads to the fact that when $0 < \beta \leq 3/(\tau + 1)$, the positive equilibrium 1/2 of the genotype selection model

$$y_{n+1} = \frac{y_n e^{\beta(1-2y_{n-\tau})}}{1 - y_n + y_n e^{\beta(1-2y_{n-\tau})}}$$

is a global attractor for all solutions originated from positive initial conditions. Our result matches the computational result $0 < \beta \leq 4\cos(\tau\pi/(2\tau + 1))$ suggested in [**3**].

1. Introduction. In [**1**, pp. 513–563], May proposed a genotype selection model of the form

$$y_{n+1} = \frac{y_n e^{\beta(1-2y_n)}}{1 - y_n + y_n e^{\beta(1-2y_n)}}, \quad n = 0, 1, 2, \dots ,$$

and investigated the local stability of the equilibrium solution $\{y_n\} = \{1/2\}$. Later in [**2**], Grove et al. showed that the equilibrium 1/2 is globally asymptotically stable if $0 < \beta \leq 4$, and unstable if $\beta > 4$. In the same paper [**2**], a positive integer delay τ is introduced into the above model to form

$$(1.1) \qquad y_{n+1} = \frac{y_n e^{\beta(1-2y_{n-\tau})}}{1 - y_n + y_n e^{\beta(1-2y_{n-\tau})}}, \quad n = 0, 1, 2, \dots ,$$

and it is shown that the equilibrium 1/2 is locally asymptotically stable if $0 < \beta < 4\cos(\tau\pi/(2\tau + 1))$, and unstable if $\beta > 4\cos(\tau\pi/(2\tau + 1))$.

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