ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 33, Number 3, Fall 2003

INFLECTION POINTS AND NONSINGULAR EMBEDDINGS OF SURFACES IN $\mathbf{R}^5$

D.K.H. MOCHIDA, M.C. ROMERO-FUSTER AND M.A.S. RUAS

ABSTRACT. We define asymptotic direction fields on surfaces embedded in $\mathbf{R}^5$ and characterize their critical points both as umbilics of height functions and as singular points of order 2 of the embedding in Feldman's sense. We show that there are at least one and at most five of these fields defined locally at each point of a generically embedded closed surface. We use this viewpoint in order to consider the existence of singular points of order 2 on a given surface.

1. Introduction. The osculating space of order k at a point p of a m-dimensional manifold M in $\mathbf{R}^n$ is the linear subspace $T_p^k M$ spanned by the osculating k-spaces of all the curves contained in M passing through p. A smooth map $f : M \to N$ between smooth manifolds M and N is said to be **nondegenerate** or **non singular of order k** if it induces an injective linear map $T_p^k f : T_p^k M \to T_{f(p)}^k N$, $\forall p \in M$. These maps were studied by E.A. Feldman ([**5**]–[**7**]), who determined the dimensions m, n of the manifolds M and N for which the set of non degenerate embeddings of order k is open and dense in the set of all the embeddings of M in N with the Whitney C^∞-topology and developed several geometrical applications of these methods.

The existence of nondegenerate embeddings of order k from M to N appears to be related to the global geometry of these manifolds. An interesting question arising in this context is that of which surfaces admit nondegenerate embeddings of order 2 in $\mathbf{R}^n$. For this question to make sense we must consider $n = 5, 6$, for when $n < 5$ there are no such maps, and for $n > 6$, Feldman proved that they form a dense set in $Emb(M, \mathbf{R}^n)$. We consider here the case $n = 5$. To approach this problem we use the family of height functions induced by an embedding

Work of the first author partially supported by FAPESP grant no. 1997/13335-6.
Work of the second author partially supported by DGCYT grant no. BFM2000-1110.
Work of the third author partially supported by FAPESP grant no. 97/10735-3 and CNPq grant no. 300066/88-03.
Received by the editors on October 6, 2000, and in revised form on February 15, 2001.