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THE DIRICHLET PROBLEM FOR QUASIMONOTONE SYSTEMS OF SECOND ORDER EQUATIONS

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ABSTRACT. We prove the existence of a solution of the Dirichlet problem $u'' + f(t,u) = 0$, $u(0) = u(1) = 0$ between upper and lower solutions, where $f : [0,1] \times E \to E$ is quasimonotone increasing in its second variable with respect to a general solid cone.

1. Introduction. Let E be a finite-dimensional real vector space ordered by a cone K. A cone K is a nonempty closed convex subset of E with $\lambda K \subseteq K$ $(\lambda \geq 0)$, and $K \cap (-K) = \{0\}$. As usual, $x \leq y :\Longleftrightarrow y - x \in K$. Furthermore we assume that K is solid, that is, $K^0 \neq \varnothing$, and we write $x \ll y$ if $y - x \in K^0$. For $x \leq y$ let $[x, y]$ denote the order interval of all z with $x \leq z \leq y$. Let K^* denote the dual cone of K, that is, the set of all $\varphi \in E^*$ with $\varphi(x) \geq 0$ $(x \geq 0)$. We fix $p \in K^0$ and consider E to be normed by $\|\cdot\|$, the Minkowski functional of $[-p, p]$. Note that $-\|x\|p \leq x \leq \|x\|p$, $x \in E$.

A function $g : E \to E$ is called quasimonotone increasing (qmi for short), in the sense of Volkmann [**16**], if

$$x, y \in E, \quad x \leq y, \quad \varphi \in K^*, \quad \varphi(x) = \varphi(y) \Longrightarrow \varphi(g(x)) \leq \varphi(g(y)).$$

A function $f : [0,1] \times E \to E$ is called qmi if $x \mapsto f(t,x)$ is qmi for each $t \in [0,1]$.

In the sequel let $f : [0,1] \times E \to E$ be continuous and qmi. We consider the Dirichlet boundary value problem

$$(1) \qquad u''(t) = f(t, u(t)), \quad t \in [0,1], \quad u(0) = u(1) = 0.$$

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