ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 34, Number 1, Spring 2004

A CRITERION FOR LINEAR INDEPENDENCE OF SERIES

JAROSLAV HANČL

ABSTRACT. The paper establishes a criterion for linear independence of infinite series which consist of rational numbers. A criterion for irrationality is obtained as a consequence.

1. Introduction. There are many papers concerning the algebraic independence of infinite series. Among them we can cite Töpfer [**14**], Loxton and Poorten [**11**] and Kubota [**10**]. A nice survey of results of this kind can be found in the book of Nishioka [**12**].

Other results of this nature include the linear independence of logarithms of special rational numbers which can be found in Sorokin [**13**] and Bezivin's result in [**3**] which proves linear independence of roots of special functional equations.

A special case of linear independence is irrationality. In [**1**] Badea proved the following theorem.

Theorem 1.1. *Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be two sequences of positive integers such that, for every large n,*

$$a_{n+1} > \frac{b_{n+1}}{b_n}\, a_n^2 - \frac{b_{n+1}}{b_n} a_n + 1.$$

Then the series $\sum_{n=1}^{\infty} \frac{b_n}{a_n}$ is an irrational number.

This result is improved in [**2**]. Another criterion of irrationality was proved by Duverney in [**6**]. In 1992 in [**4**] Borwein proved that the series $\sum_{n=1}^{\infty} \frac{1}{q^n+r}$ is irrational and not Liouville whenever q is an integer ($q \neq 0, \pm 1$) and r is a nonzero rational number ($r \neq q^n$). The same author together with Zhou in [**5**] proved the following theorem.

AMS Mathematics Subject Classification. 11J72.
Supported by grants no. 201/01/0471 and no. 201/04/0393 of the Czech Grant Agency.