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SPECTRAL ESTIMATES FOR THE COMMUTATOR OF TWO-DIMENSIONAL HILBERT TRANSFORMATION AND THE OPERATOR OF MULTIPLICATION WITH A C^1 FUNCTION

M.R. DOSTANIĆ

1. Introduction and notation. Let Γ be a set of simple non-intersecting closed contours of Lyapunov type and S_Γ be the singular integral along Γ:

$$(S_\Gamma \varphi)(t) = \frac{1}{\pi i} \int_\Gamma \frac{\varphi(s)}{s-t}\, ds, \quad t \in \Gamma;$$

the contour Γ is considered oriented in some manner. It is well known that the operator S_Γ is bounded in each of the spaces $L^p(\Gamma)$, $1 < p < \infty$.

Also, it is known that if a function $t \mapsto a(t)$ satisfies a Hölder condition on Γ or if $a \in C(\Gamma)$, then the operator $aS_\Gamma - S_\Gamma a$ is compact on $L^p(\Gamma)$, see [**6**]. In the special case, when Γ is an interval on the real axis, S_Γ is the Hilbert transformation.

Instead of S_Γ it is possible to consider some other singular integral operator and study its spectral properties.

Let Ω be a domain in $\mathbf{C}$. Denote by $L^2(\Omega)$ the space of complex-valued functions on Ω such that the norm

$$\|f\| = \left(\int_\Omega |f(\xi)|^2 \, dA(\xi) \right)^{1/2}$$

is finite. Here dA denotes Lebesgue measure on Ω.

It is known (see [**8**]) that the formula

$$H_\Omega f(z) = -\frac{1}{\pi} \text{ p.v. } \int_\Omega \frac{f(\xi)}{(\xi - z)^2}\, dA(\xi)$$

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