ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 34, Number 1, Spring 2004

A NEW CLASS OF NORMED SPACES WITH NONTRIVIAL GROUPS OF ISOMETRIES AND SOME ESTIMATES FOR OPERATORS WITH GIVEN ACTION

B.L. CHALMERS AND M.I. OSTROVSKII

ABSTRACT. We introduce and study a new class of finite-dimensional normed spaces with non-trivial groups of isometries. We call it the class of spaces with *distinguished bases*. The introduction of this new class is motivated by the following: (a) this class is a natural class that contains both the class of spaces with unconditional bases and the class of spaces spanned by characters in translation invariant function spaces on compact abelian groups; (b) there is a very simple formula for 2-summing norms of the operators that are diagonal with respect to distinguished bases.

An additional motivation for introduction of the notion of a distinguished basis is its relations with the problem of the estimate of norms of operators with given action. More precisely, let A be a $k \times k$ matrix and let V be a k-dimensional normed space. The set of all operators on V whose matrix with respect to some basis in V is A is denoted by $L_A(V)$. The problem is to estimate $\inf\{||T|| : T \in L_A(V)\}$, where $||\cdot||$ is one of the natural norms on the space of linear operators on V. The second part of the paper is devoted to some aspects of this general problem.

1. Introduction. Different classes of spaces with nontrivial groups of isometries play an important role in Banach Space Theory and in Asymptotic Theory of Finite-Dimensional Normed Spaces (see, for example, [**41**]). Three of such classes have been extensively studied. These are the classes of (1) spaces with unconditional bases, (2) spaces with symmetric bases, and (3) spaces with enough symmetries. The purpose of this paper is to introduce one more class, we call it the class of spaces with *distinguished bases*. The introduction of this new class is motivated by the following: (a) this class is a natural class that contains both the class of spaces with unconditional bases and the class of spaces spanned by characters in translation invariant function spaces

2000 AMS *Mathematics Subject Classification.* Primary 52A21, 46B07.
Received by the editors on April 27, 2001, and in revised form on February 7, 2002.