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SPECTRAL PROPERTIES OF THE NONHOMOGENEOUS KLEIN-GORDON s-WAVE EQUATIONS

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ABSTRACT. In this article we investigate the eigenvalues and the spectral singularities of the boundary value problem

$$y'' + [\lambda - p(x)]^2 y = f(x), \quad x \in \mathbf{R}_+ = [0, \infty),$$
$$\alpha y'(0) - \beta y(0) = 0,$$

in the space $L_2(\mathbf{R}_+)$ where p and f are complex-valued functions and $\alpha, \beta \in \mathbf{C}$, with $|\alpha| + |\beta| \neq 0$.

1. Introduction. Let L denote the operator generated in $L_2(\mathbf{R}_+)$ by the differential expression

$$l(y) = -y'' + q(x)y, \quad x \in \mathbf{R}_+ = [0, \infty),$$

with the boundary condition $y'(0) - hy(0) = 0$, where q is a complex-valued function and $h \in \mathbf{C}$. The study of the spectral analysis of L was begun by Naimark [**18**]. He proved that the spectrum of L consists of the eigenvalues, the continuous spectrum and the spectral singularities. The spectral singularities are poles of the kernel of the resolvent and are also imbedded in the continuous spectrum, but they are not eigenvalues.

Lyance [**16**] showed that the spectral singularities play an important role in the spectral analysis of L. He also investigated the effect of the spectral singularities in the spectral expansion. The spectral singularities of dissipative Schrödinger operators with rapidly decreasing potential were considered by Hruscev [**9**]. In [**1**] and [**15**], by means of the uniqueness theorems of analytic functions, the dependence of the structure of the spectral singularities of Quadratic Pencil of Schrödinger Operators (QPSO) was studied. A two-fold spectral expansion in terms of the principal functions of QPSO with spectral singularities has been derived in [**2**]. In that article the effect of the spectral singularities in the spectral expansion of QPSO has also been investigated via the

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