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BIFURCATIONS OF BOUNDED SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS DEPENDING ON A PARAMETER

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ABSTRACT. In this paper, using the notion of an isolated invariant set and an isolating block, an existence criterion of bifurcation points of nonstationary bounded solutions for planar systems depending on a parameter is given.

1. Introduction. Consider the one-parameter family of differential systems in R^n

$$\frac{dx}{dt} = F(x, \lambda). \tag{1.1}$$

Let $F : R^n \times R \to R^n$ be continuous and assume that, for each $\lambda \in R$, the solution of an initial value problem is unique.

Each zero of F is called a stationary solution of (1.1). Clearly, if (x_0, λ_0) satisfies $F(x_0, \lambda_0) = 0$, then x_0 is a critical point of the $\lambda = \lambda_0$ system (1.1). In this paper we shall investigate bifurcation points of nonstationary bounded solutions of (1.1), where a bounded solution means that it is bounded both in the forward and backward time directions.

Definition 1.1 [**3**]. A point $(x_0, \lambda_0) \in R^n \times R$ is said to be a bifurcation point of nonstationary bounded solutions of the system (1.1) if for any open neighborhood U of (x_0, λ_0) there is a nonstationary solution of (1.1) included in U.

It follows directly from Definition 1.1 that if (x_0, λ_0) is a bifurcation point, then x_0 has to be a critical point of the $\lambda = \lambda_0$ system (1.1).

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