HURWITZ SPACES AND
BRAID GROUP REPRESENTATIONS

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ABSTRACT. We give a new construction of a Hurwitz space, which is a moduli space of all branched covers of the Riemann sphere having a given combinatorial description. The action of the fundamental group of the Hurwitz space on the homology of the branched cover gives rise to a linear represenation of a finite index subgroup of the spherical braid group, or of a finite extension of such a subgroup. We construct examples of each of these two cases. Using a result of Fried, we use these representations to extract information about the dimension of the image of the Hurwitz space in the genus g moduli space.

0. Introduction. In this paper we investigate certain moduli spaces, *Hurwitz spaces*, of branched covers of the Riemann sphere S^2 and representations of finite index subgroups of the spherical braid group which arise from these Hurwitz spaces. (By *spherical* braid group, we mean the group of braids in the two-sphere; we will refer to the more classical group of braids in the plane as the *planar* braid group.)

Hurwitz spaces play an important role in realizing groups as Galois groups, a role which has been explored primarily by Fried and Völklein in [**4, 5, 7, 14, 15**], etc. In these works they have given a couple of constructions of Hurwitz spaces, examined their algebraic structure, and explored their applications to the inverse Galois problem. In Section 1 of this paper we give an alternative construction of the Hurwitz spaces, exhibiting them as homogeneous spaces of $\mathrm{Aut}\,(S^2)$, the group of orientation preserving homeomorphisms of S^2. This point of view enables us to prove that the universal cover of a Hurwitz space is homotopy equivalent to S^3 (see the discussion just after Proposition 4), which is equivalent to showing that the Teichmuller space of a sphere