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PERTURBATIONS OF p-ADIC LINEAR OPERATORS

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ABSTRACT. In this paper we give the perturbation theory for p-adic continuous linear operators. In particular, we deal with the gap between ranges of linear operators, the gap between kernels of linear operators and the gap of the solution sets of linear operator equations.

1. Introduction. The problems of perturbations of p-adic linear operators have been studied by many authors. Many of them dealt with the perturbations of the index of linear operators, cf. [**1, 8, 9**]. In this paper we deal with the perturbation of the gaps between the closed convex subsets which are defined by using the continuous linear operators with a generalized inverse.

Let E and F be non-Archimedean Banach spaces, let T and A be continuous linear operators from E to F, and let $R(T)$ be closed. Let b and $\bar{b}$ be fixed elements of $R(T)$ and $R(T+A)$, respectively, and set $X(T,b) = \{x \in E : Tx = b\}$. If T has a generalized inverse S, then under some conditions we show that the gap between $R(T)$ and $\overline{R(T+A)}$ is estimated by $\|S\|\,\|A\|$, the gap between $\operatorname{Ker}(T)$ and $\operatorname{Ker}(T+A)$ is estimated by $\|SA\|$ and the gap between $X(T,b)$ and $X(T+A,\bar{b})$ is also estimated by $\|SA\|$.

2. Preliminaries. Throughout, K is a non-Archimedean valued field that is complete under the metric induced by the nontrivial valuation $|\cdot|$ and E, F are Banach spaces over K. Let $L(E,F)$ denote the set of all continuous linear operators from E to F. For $B \in L(E,F)$, $R(B)$ and $\operatorname{Ker}(B)$ are the range and the kernel of B, respectively. If M is a linear subspace of E, $B \mid M$ is the restriction of B to M. The identity map on E is denoted by I_E. A subset V of E is said to be convex if, for every $x, y, z \in V$ and for every $\alpha, \beta, \gamma \in K$ with

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