ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 34, Number 4, Winter 2004

EVERY ABSOLUTELY HENSTOCK-KURZWEIL INTEGRABLE FUNCTION IS MCSHANE INTEGRABLE: AN ALTERNATIVE PROOF

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ABSTRACT. We give an alternative proof of the well-known result that every absolutely Henstock-Kurzweil integrable function is McShane integrable.

1. Introduction. It is well known that the Lebesgue integral is a proper extension of the Riemann integral. In the late 1950s, Henstock [**4**] and Kurzweil [**6**] independently gave a slight, but ingenious, modification of the classical Riemann integral to obtain a Riemann-type definition of the Perron integral. This integral is now commonly known as the Henstock-Kurzweil integral [**9, 12**], the Kurzweil-Henstock integral [**8, 14**], the gauge integral [**13**] or the Henstock integral [**1, 3, 7**], and we shall use the term "Henstock-Kurzweil integral." Later, McShane [**10**] modified the Henstock-Kurzweil integral to yield a Riemann-type definition of the Lebesgue integral, which is also commonly referred to as the McShane integral [**1, 3, 7, 8, 12**–**14**]. It turns out that f and $|f|$ are both Henstock-Kurzweil integrable on a compact subinterval E of the real line if and only if f is McShane integrable there. In 1980 Pfeffer in [**11**, p. 46] proposed a problem to prove, using only the definitions of Henstock-Kurzweil and McShane integrals, that absolutely Henstock-Kurzweil integrable functions are McShane integrable. Since then a fairly large number of proofs have been offered. See, for example, [**1, 3, 7, 8, 13, 14**]. However, their proofs either involve convergence theorems or the existing techniques rely heavily on the real-valued property of integrable functions. In this paper we give an alternative proof of the above result which is also valid for Banach-valued integrable functions satisfying the Saks-Henstock lemma. Moreover our method, unlike the existing known proofs, uses neither the measurability of the integrand nor convergence theorems.

Received by the editors on July 8, 2002, and in revised form on September 5, 2002.